数学演習 B 問題 (解析 1B) No.5(略解)

5-1. (1) $\displaystyle\int_0^1 e^{2y}\Big\{\int_0^2 e^{3x}dx\Big\}dy=\int_0^1 e^{2y}\frac{e^6-1}{3}dy$ 　　または

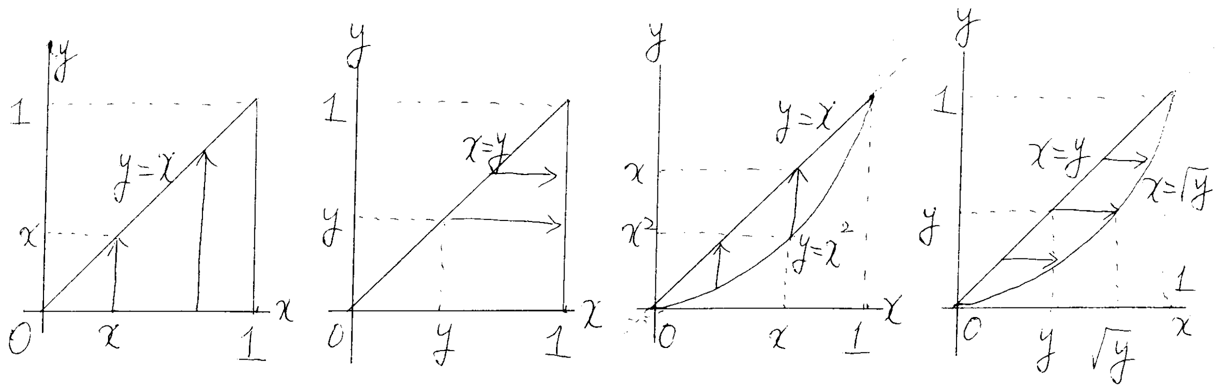
$\displaystyle\int_0^2 e^{3x}\Big\{\int_0^1 e^{2y}dy\Big\}dx=\int_0^2 e^{3x}\frac{e^2-1}{2}dx$ より $\displaystyle\frac{e^8-e^6-e^2+1}{6}$.

(2) $\displaystyle\int_0^{\frac{\pi}{2}}\Big\{\int_0^{\frac{\pi}{2}}\cos(x+y)dy\Big\}dx=\int_0^{\frac{\pi}{2}}\Big[\sin(x+y)\Big]_{y=0}^{\frac{\pi}{2}}dx=\int_0^{\frac{\pi}{2}}(\cos x-\sin x)dx=0.$

(3) $\displaystyle\int_0^4\Big\{\int_0^1(x+y)^2dy\Big\}dx=\int_0^4\Big[\frac{1}{3}(x+y)^3\Big]_{y=0}^1dx=\int_0^4\frac{1}{3}\Big((x+1)^3-x^3\Big)dx=\frac{1}{3}\times\frac{1}{4}\Big[(x+1)^4-x^4\Big]_{x=0}^4=\frac{92}{3}.$

または, $\displaystyle\int_0^1\Big\{\int_0^4(x+y)^2dx\Big\}dy=\int_0^1\Big[\frac{1}{3}(x+y)^3\Big]_{x=0}^4dy=\int_0^1\frac{1}{3}\Big((4+y)^3-y^3\Big)dy=\frac{1}{3}\times\frac{1}{4}\Big[(4+y)^4-y^4\Big]_{y=0}^1=\frac{92}{3}.$

5-2.



(1) $D_1=\{(x,y)\mid 0\leqq x\leqq 1,\ 0\leqq y\leqq x\}=\{(x,y)\mid 0\leqq y\leqq 1,\ y\leqq x\leqq 1\}$

$D_2=\{(x,y)\mid 0\leqq x\leqq 1,\ x^2\leqq y\leqq x\}=\{(x,y)\mid 0\leqq y\leqq 1,\ y\leqq x\leqq \sqrt{y}\}$

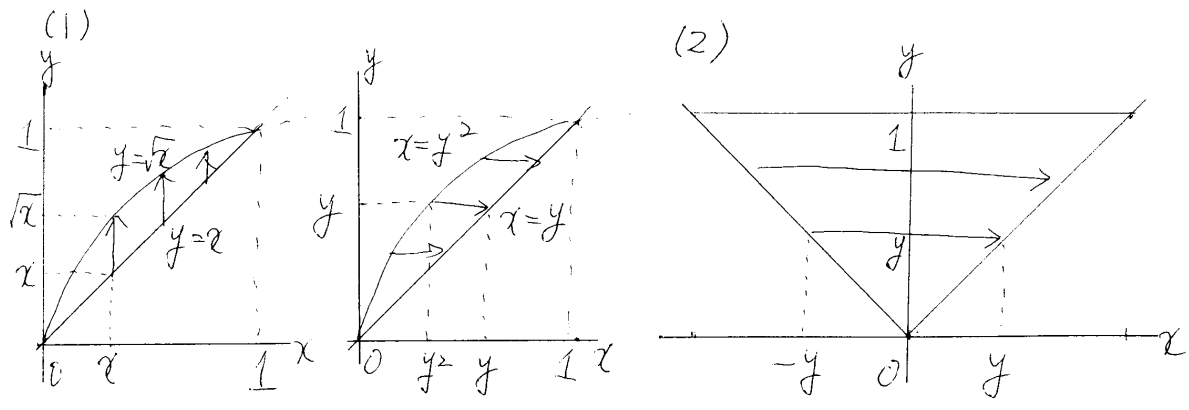
(2) (i) $\displaystyle\iint_{D_1}ydxdy=\int_0^1\Big\{\int_0^x ydy\Big\}dx=\int_0^1\frac{1}{2}x^2dx=\frac{1}{6}$ または

$\displaystyle\iint_{D_1}ydxdy=\int_0^1\Big\{\int_y^1 ydx\Big\}dy=\int_0^1 y(1-y)dy=\frac{1}{6}.$

(ii) $\displaystyle\iint_{D_2}ydxdy=\int_0^1\Big\{\int_{x^2}^x ydy\Big\}dx=\int_0^1\frac{1}{2}(x^2-x^4)dx=\frac{1}{15}$ または

$\displaystyle\iint_{D_2}ydxdy=\int_0^1\Big\{\int_y^{\sqrt{y}} ydx\Big\}dy=\int_0^1 y(\sqrt{y}-y)dy=\frac{1}{15}.$

5-3.



(1) D_1 をたて線集合 $D_1=\{(x,y)\mid 0\leqq x\leqq 1, x\leqq y\leqq\sqrt{x}\}$ と考えると

$$\iint_{D_1}(x+y)dxdy=\int_0^1\Big\{\int_x^{\sqrt{x}}(x+y)dy\Big\}dx=\int_0^1\Big\{x(\sqrt{x}-x)+\Big[\frac{1}{2}y^2\Big]_{y=x}^{\sqrt{x}}\Big\}dx=\int_0^1\Big(\frac{1}{2}x+x^{\frac{3}{2}}-\frac{3}{2}x^2\Big)dx=\frac{3}{20}.$$

また，横線集合 $D_1=\{(x,y)\mid 0\leqq y\leqq 1, y^2\leqq x\leqq y\}$ と考えると

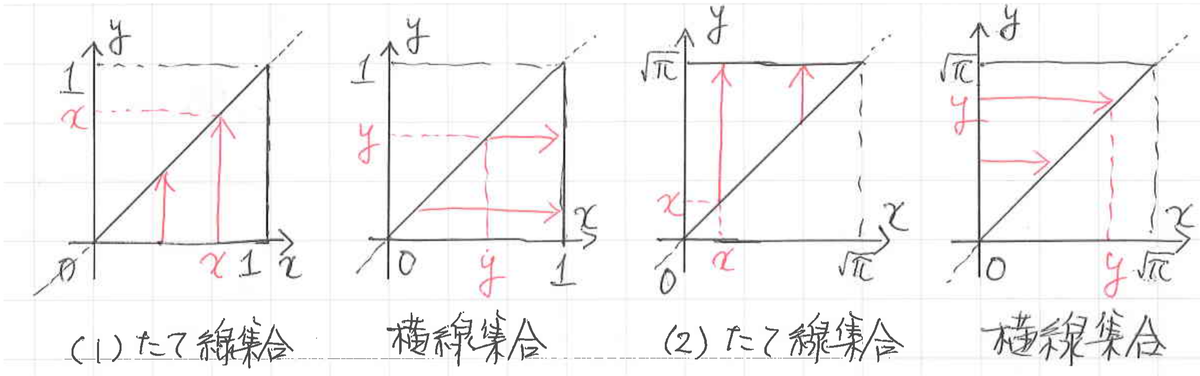
$$\iint_{D_1}(x+y)dxdy=\int_0^1\Big\{\int_{y^2}^{y}(x+y)dx\Big\}dy=\int_0^1\Big(\frac{1}{2}(y^2-y^4)+y(y-y^2)\Big)dy=\int_0^1\Big(\frac{3}{2}y^2-y^3-\frac{1}{2}y^4\Big)dy=\frac{3}{20}.$$

(2) D_2 を横線集合 $D_2=\{(x,y)\mid 0\leqq y\leqq 1, -y\leqq x\leqq y\}$ と考えると，

$$\iint_{D_2}(x+y)dxdy=\int_0^1\Big\{\int_{-y}^{y}(x+y)dx\Big\}dy=\int_0^1\Big[\frac{1}{2}(x+y)^2\Big]_{x=-y}^{y}dy=\int_0^1 2y^2dy=\frac{2}{3}.$$

D_2 をたて線集合で 1 つの形で書くなら，$D_2=\{(x,y)\mid -1\leqq x\leqq 1,\ |x|\leqq y\leqq 1\}$ である．D_2 上の積分をたて線集合にこだわって考えるなら，$x\leqq 0$ の部分と $x\geqq 0$ の部分に分けて考えることになる．

5-4.



(1) 横線集合と考えると $\displaystyle\int_0^1\Big\{\int_y^1 e^{x^2}dx\Big\}dy$ となって積分できないが,
たて線集合 $\{(x,y)\mid 0\leqq x\leqq 1,\ 0\leqq y\leqq x\}$ と考えると

$$\iint_{0\leqq y\leqq x\leqq 1}e^{x^2}dxdy=\int_0^1\Big\{\int_0^x e^{x^2}dy\Big\}dx=\int_0^1 xe^{x^2}dx=\frac{e-1}{2}.$$

(2) たて線集合と考えると $\displaystyle\int_0^{\sqrt{\pi}}\Big\{\int_x^{\sqrt{\pi}}\sin(y^2)dy\Big\}dx$ となって積分できないが,
横線集合 $\{(x,y)\mid 0\leqq y\leqq\sqrt{\pi},\ 0\leqq x\leqq y\}$ と考えると

$$\iint_{0\leqq x\leqq y\leqq\sqrt{\pi}}\sin(y^2)dxdy=\int_0^{\sqrt{\pi}}\Big\{\int_0^y\sin(y^2)dx\Big\}dy=\int_0^{\sqrt{\pi}}y\sin(y^2)dy=1.$$

5-5. $\displaystyle I(n)=\int_0^n\Big\{\int_0^x e^{-x-y}dy\Big\}dx=\int_0^n e^{-x}\Big\{\int_0^x e^{-y}dy\Big\}dx=\int_0^n e^{-x}(1-e^{-x})dx=\frac{1}{2}-e^{-n}+\frac{1}{2}e^{-2n}\to\frac{1}{2}.$

5-6. $\displaystyle J(a)=\int_a^1\Big\{\int_0^y(\frac{x}{y})^4dx\Big\}dy=\int_a^1\frac{1}{y^4}\Big[\frac{1}{5}x^5\Big]_{x=0}^{y}dy=\int_a^1\frac{1}{5}ydy=\frac{1}{10}(1-a^2)\to\frac{1}{10}\qquad(a\to 0).$