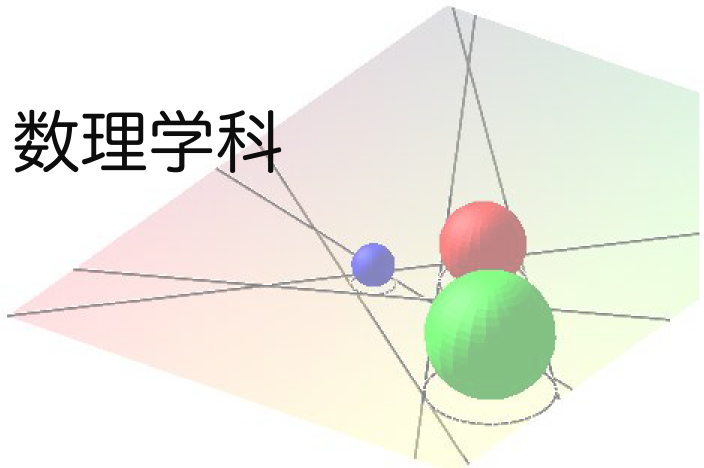
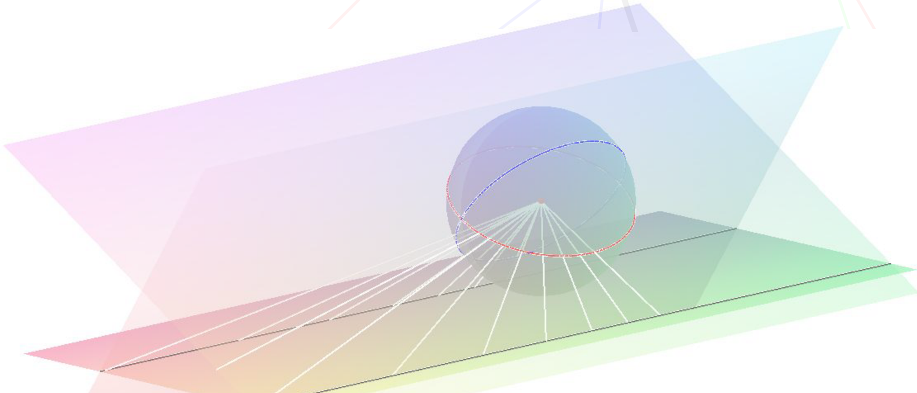
広島大学附属高等学校　数学科
フロンティアサイエンス講義（FS講義）

一目でわかる証明
ーVisual Proofsー
I

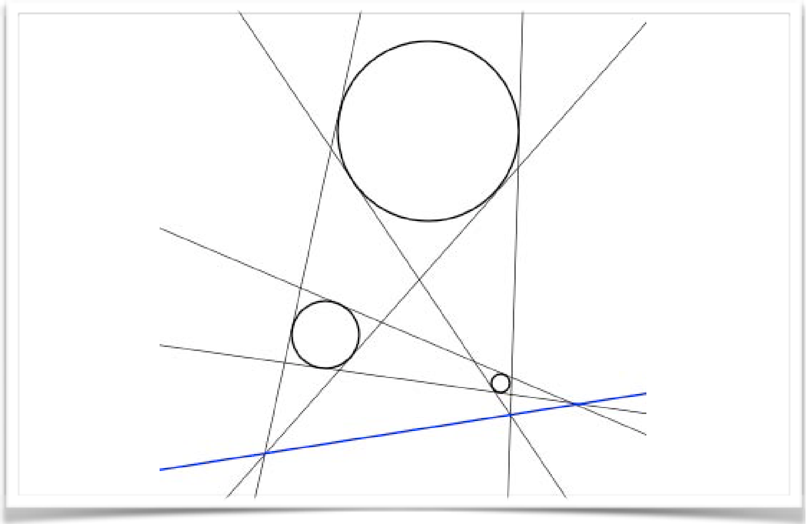


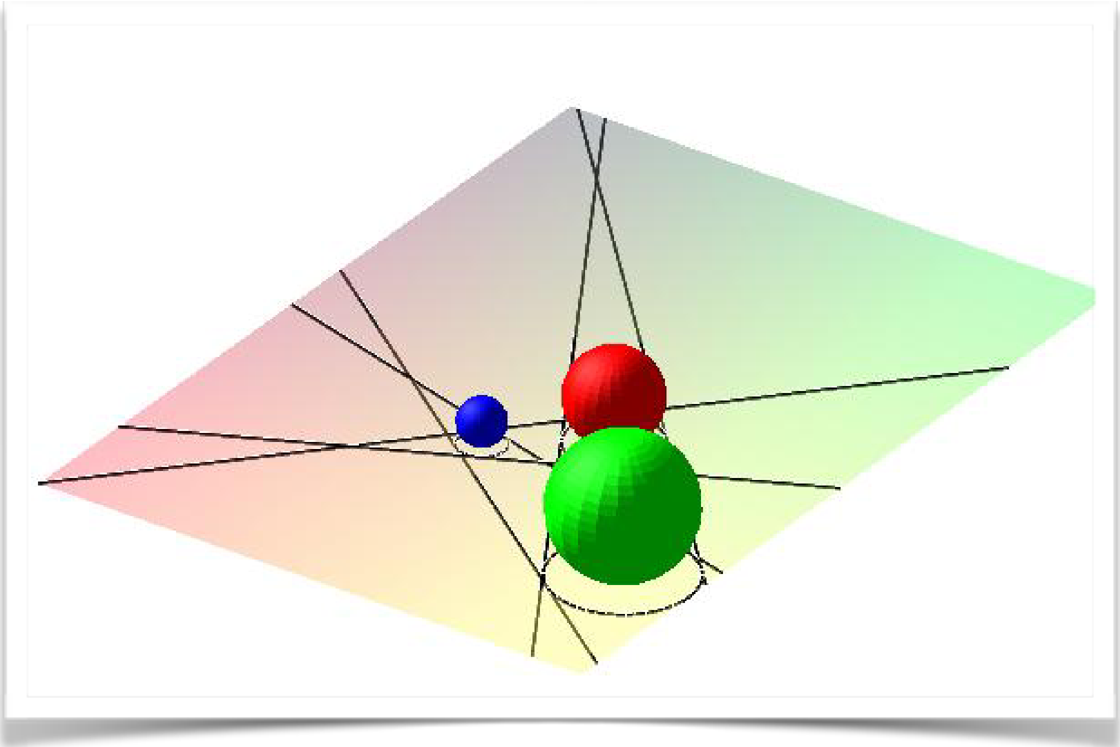
青山学院大学　理工学部　物理・数理学科
西山 享

2016.06.01



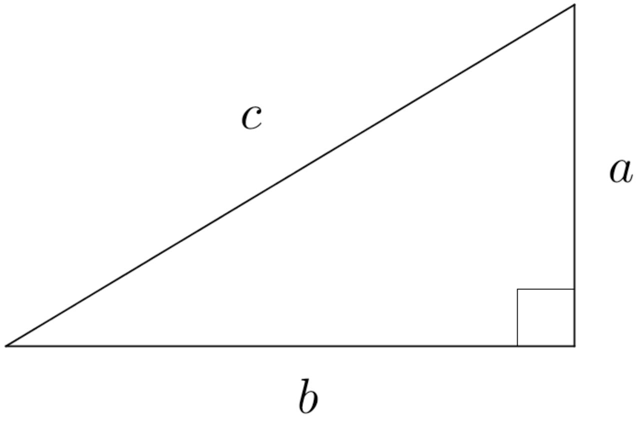
Part 1
ウォーミングアップ

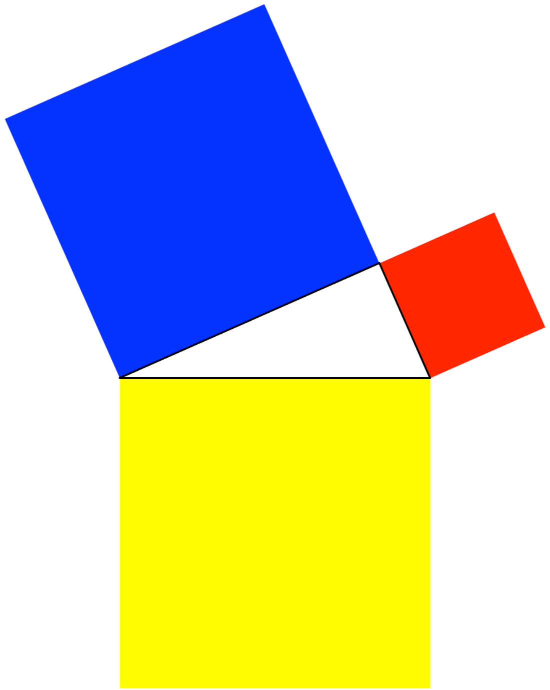




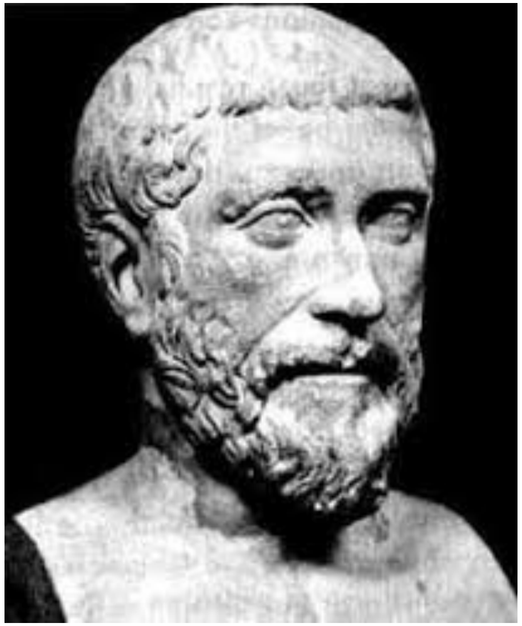
ピタゴラスの定理

$$a^2 + b^2 = c^2$$



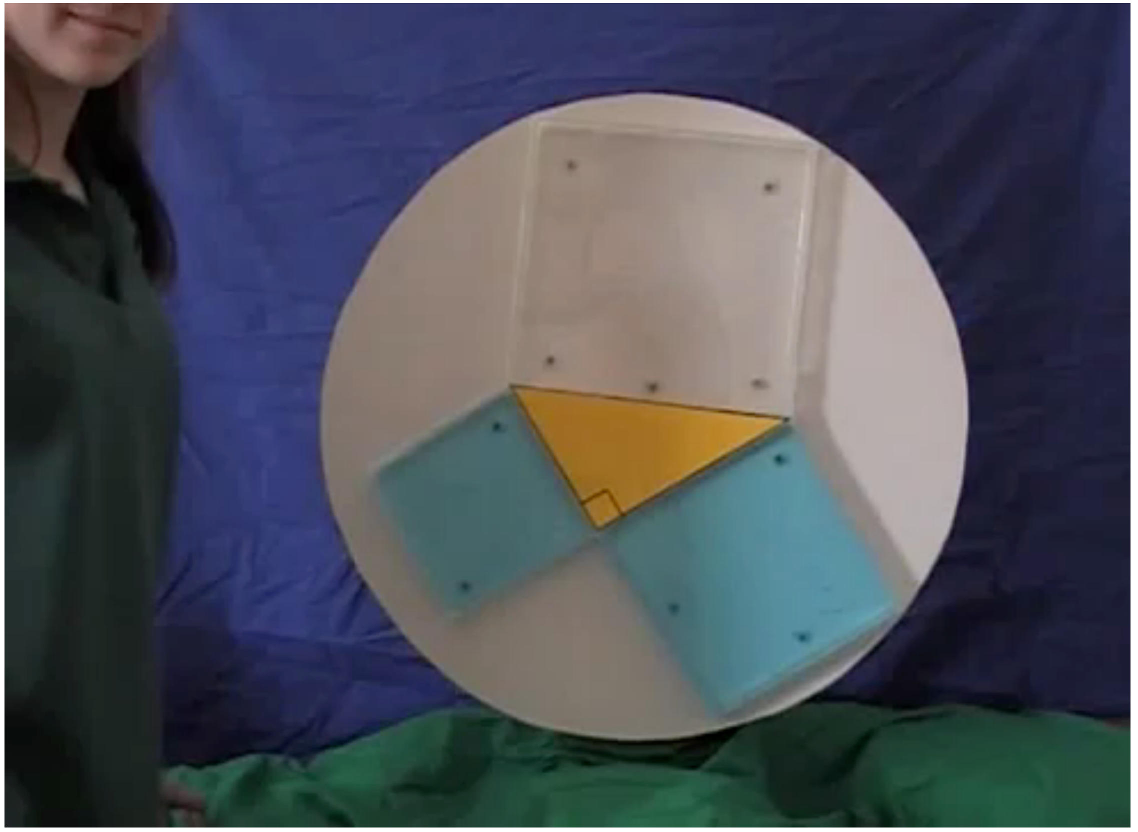


「万物は数である」



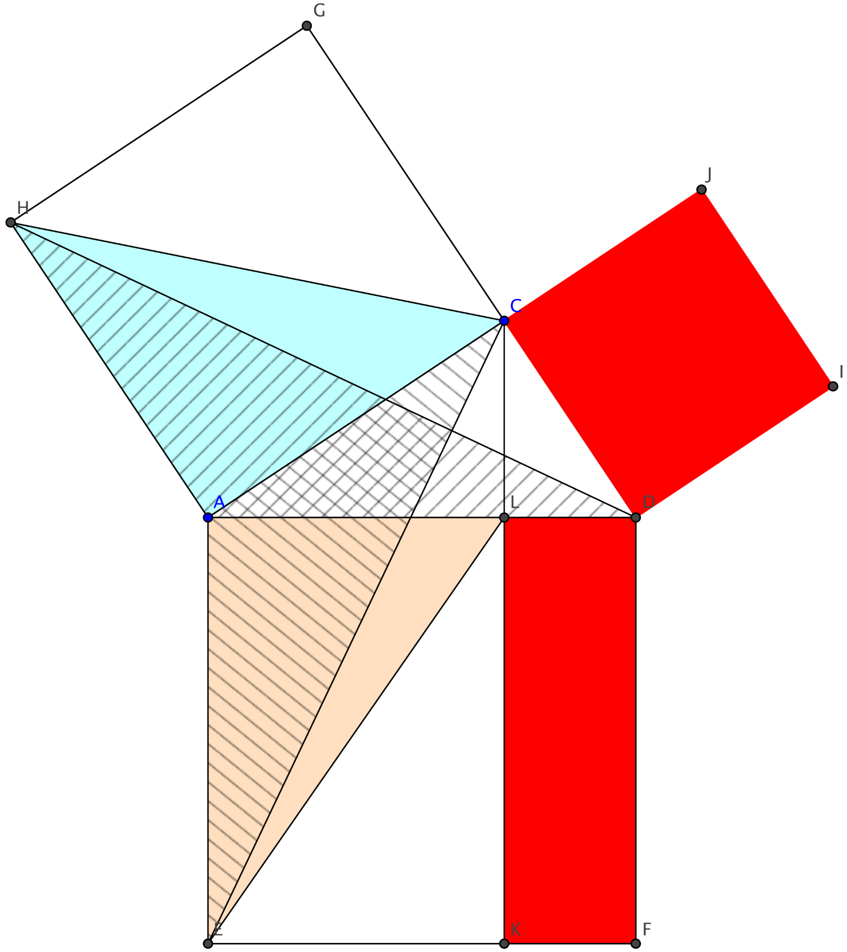
サモアのピタゴラス (前570??—前500??)

証明!!??



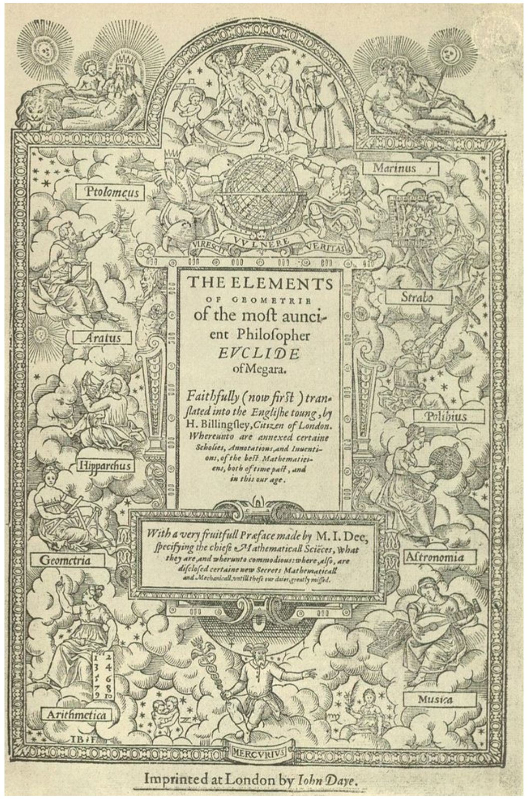
- Pythagorean theorem water demo
- http://www.youtube.com/watch?v=CAkMUdeB06o

ユークリッドの証明



ユークリッド「原論」
(The Elements)

アレキサンドリアのユークリッド
プトレマイオス１世の頃 (前306-283)



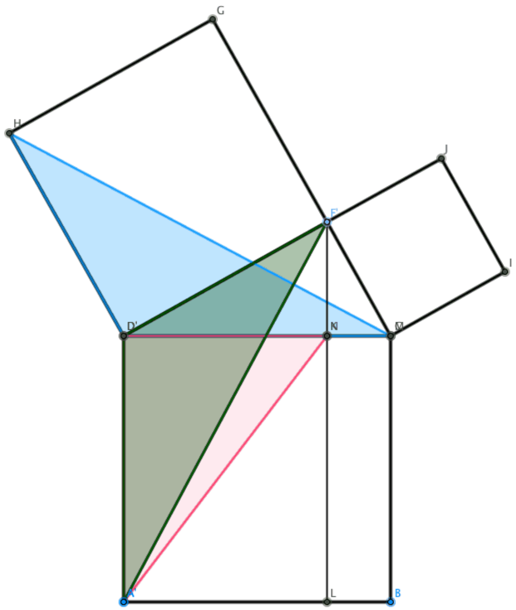
THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

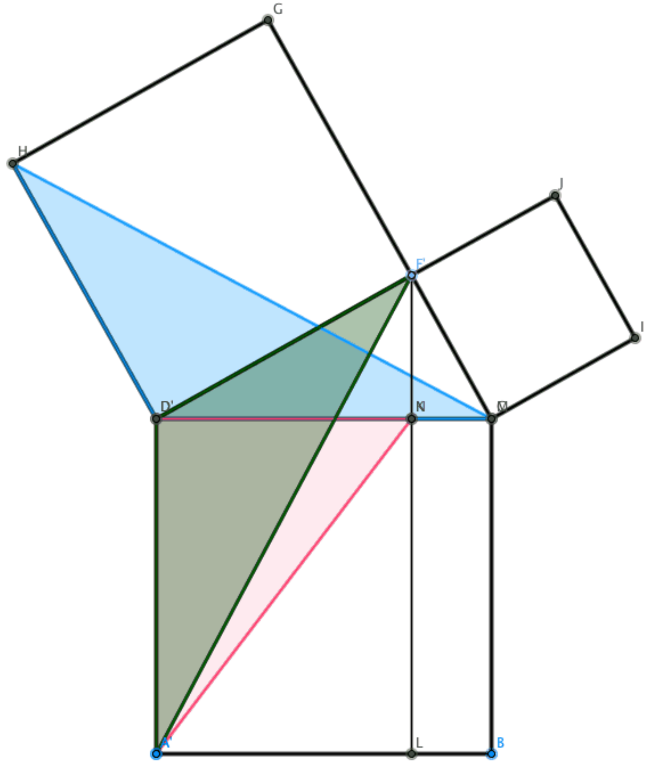
Faithfully (now first) tran-
slated into the Englishe toung, by
H. Billingsley, Citizen of London.
Whereunto are annexed certaine
Scholies, Annotations, and Inventi-
ons, of the best Mathematici-
ens, both of time past, and
in this our age.

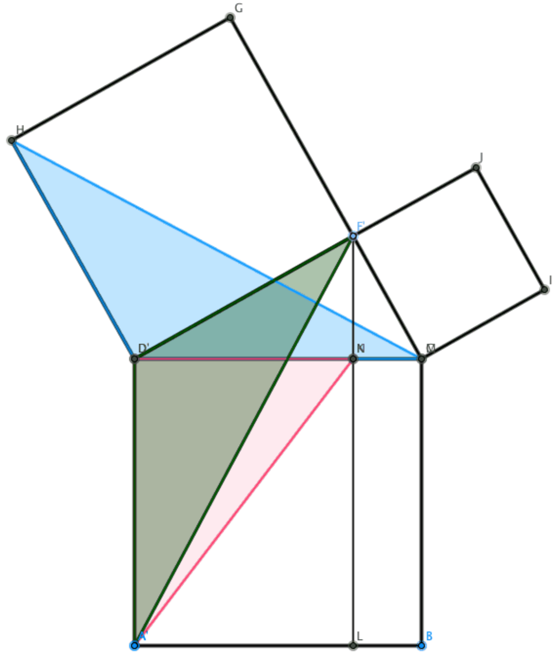
With a very fruitfull Præface made by M. I. Dee,
specifying the chiefe Mathematicall Sciēces, what
they are, and wherunto commodious: where, also, are
disclosed certaine new Secrets Mathematicall
and Mechanicall, vntill these our daies, greatly missed.

Imprinted at London by Iohn Daye.

ピタゴラスの証明を理解する







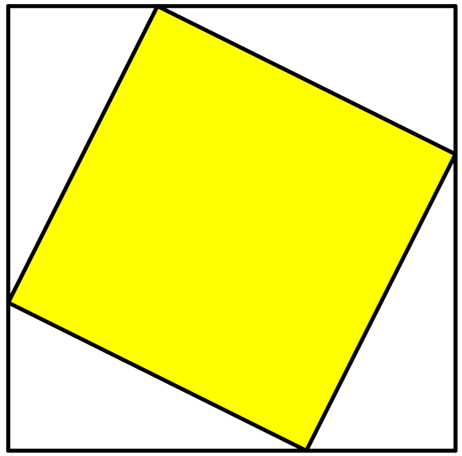
一目で分かる???証明

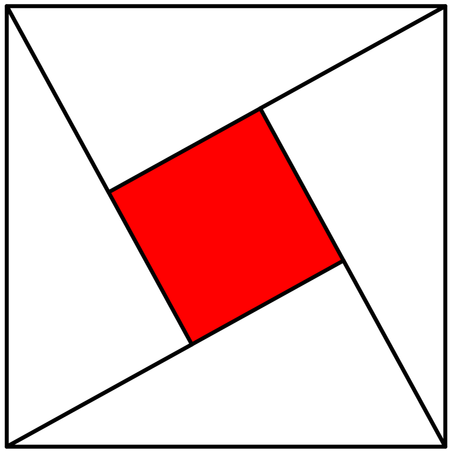
(ピタゴラスの定理)

(大正方形) = (小正方形(黄)) + 4×(直角三角形)

$$(a+b)^2 = c^2 + 4 \cdot \frac{1}{2}ab$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$





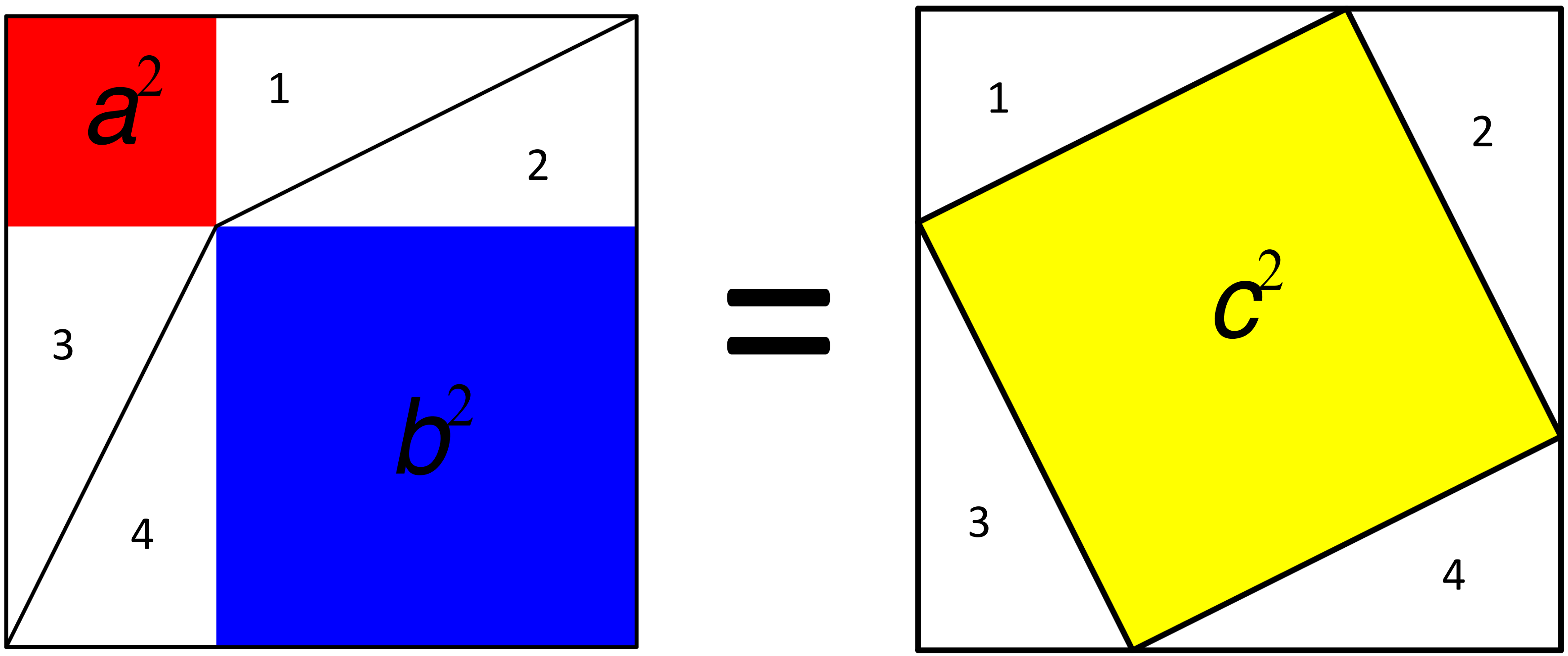
(大正方形) = (小正方形(赤)) + 4×(直角三角形)

$$c^2 = (a-b)^2 + 4 \cdot \frac{1}{2}ab$$

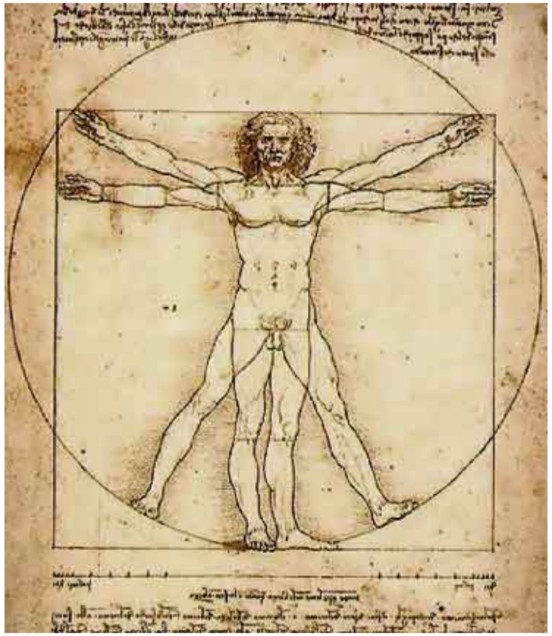
$$c^2 = a^2 - 2ab + b^2 + 2ab = a^2 + b^2$$

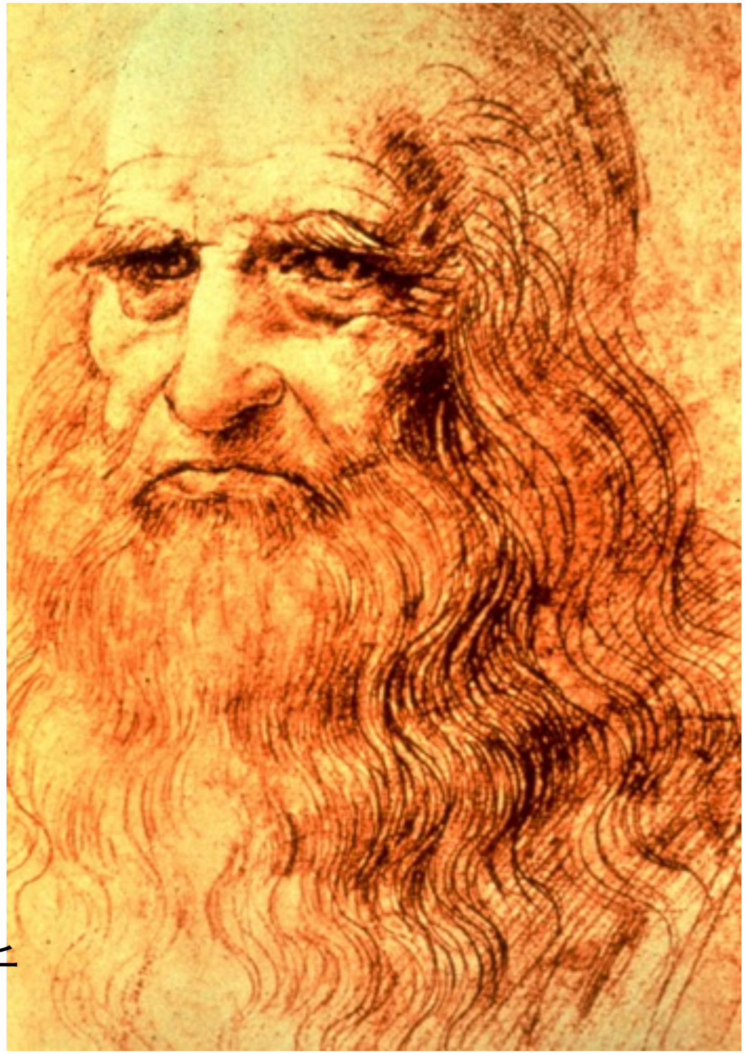
一目で分かる証明

（ピタゴラスの定理）



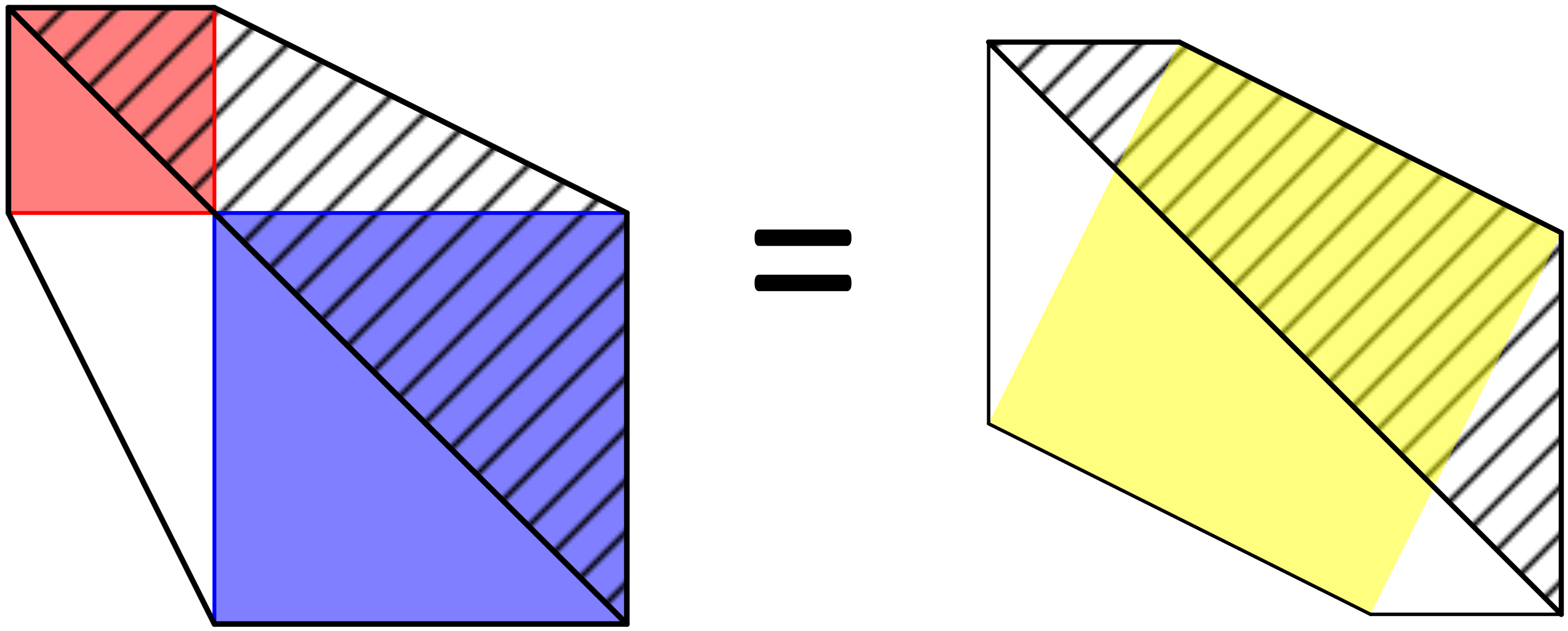
レオナルド・ダ・ヴィンチ の証明を味わう



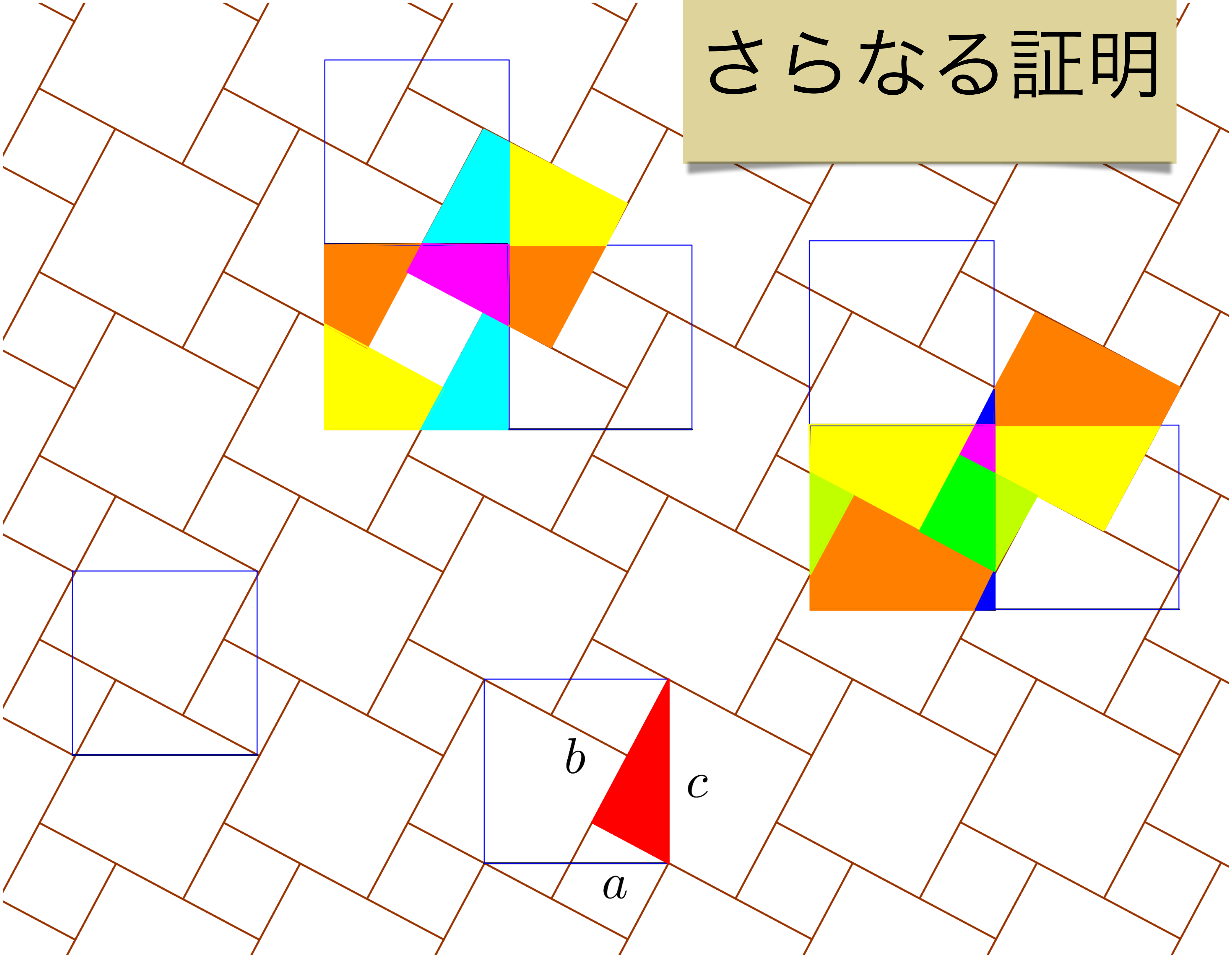


レオナルド・ダ・ヴィンチ
（1452 ー 1519）

レオナルド・ダ・ヴィンチ の証明を味わう



さらなる証明



Part 2
ベキ乗和の公式

$$1+2+3+\cdots+n=\frac{1}{2}n(n+1)$$

$$1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$$

$$1^3+2^3+3^3+\cdots+n^3=\left\{\frac{1}{2}n(n+1)\right\}^2$$

$$1^4+2^4+3^4+\cdots+n^4=\frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$$

$$1^5+2^5+3^5+\cdots+n^5=\frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$$

$1^m+2^m+3^m+\cdots+n^m\cdots$(ベルヌーイ数を使って書ける)

総和記号 $\sum$ の使い方

$\sum_{k=1}^{n}$ Summation 和

便利な使い方

$$1+2+3+\cdots+n=\sum_{k=1}^{n} k$$

$$1^2+2^2+3^2+\cdots+n^2=\sum_{k=1}^{n} k^2$$

$$\sum_{k=1}^{n} a_k = a_1+a_2+a_3+\cdots+a_n$$

$$\sum_{k=1}^{n} f(k) = f(1)+f(2)+f(3)+\cdots+f(n)$$

総和記号 $\sum$ の使い方

$\sum_{k=1}^{n}$ Summation 和

便利な使い方

$$1+2+3+\cdots+n=\sum_{k=1}^{n} k$$

$$1^2+2^2+3^2+\cdots+n^2=\sum_{k=1}^{n} k^2$$

$$\sum_{k=1}^{n} x^k = x+x^2+x^3+\cdots+x^n=\frac{x(1-x^n)}{1-x}$$

$$\sum_{k=1}^{n}\frac{1}{1+k^2}=\frac{1}{1+1^2}+\frac{1}{1+2^2}+\frac{1}{1+3^2}+\cdots+\frac{1}{1+n^2}$$

ベキ乗和（1 乗和）

$$1+2+3+\cdots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$$

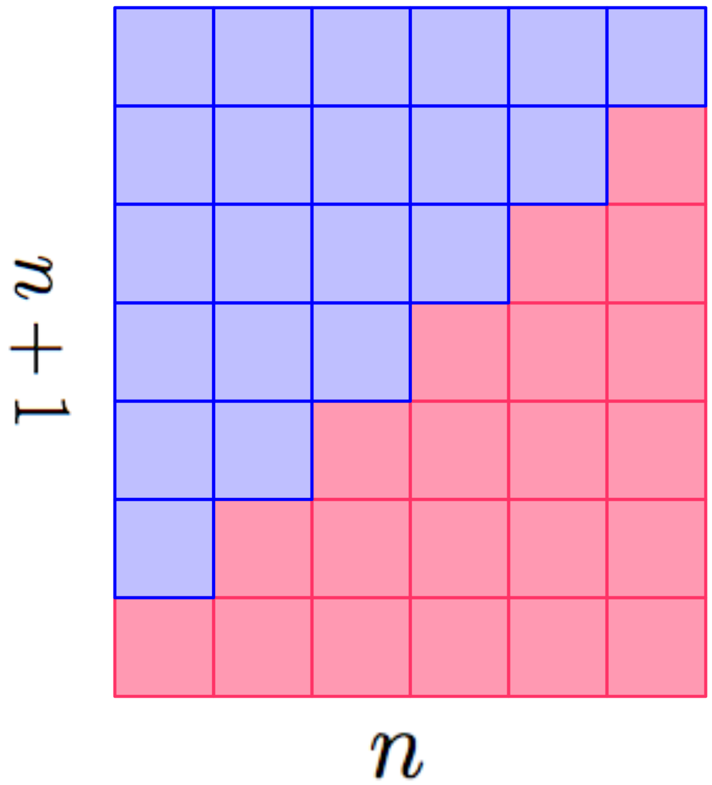
ガウス少年のひらめき

カール・フリードリッヒ・ガウス

(1777 — 1855)

ベキ乗和（1 乗和）

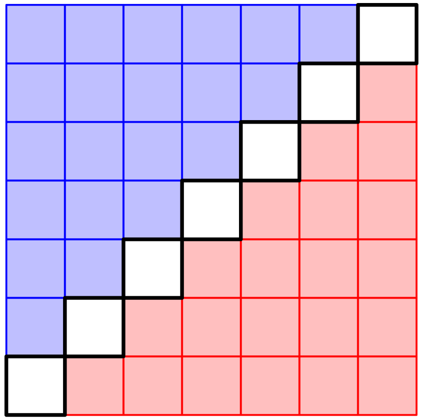
$$1+2+3+\cdots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$$



$$2\sum_{k=1}^{n}k=(\text{長方形})=n(n+1)$$

ベキ乗和（1 乗和）

$$1+2+3+\cdots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$$



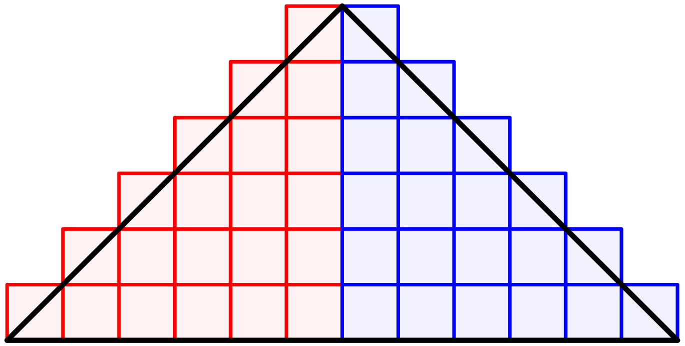
$$S:=\sum_{k=1}^{n}k$$

$$(n+1)^2=(\text{正方形})=2\,S+(\text{対角線})=2\,S+(n+1)$$

$$\therefore\quad 2\,S=(n+1)^2-(n+1)=(n+1)n$$

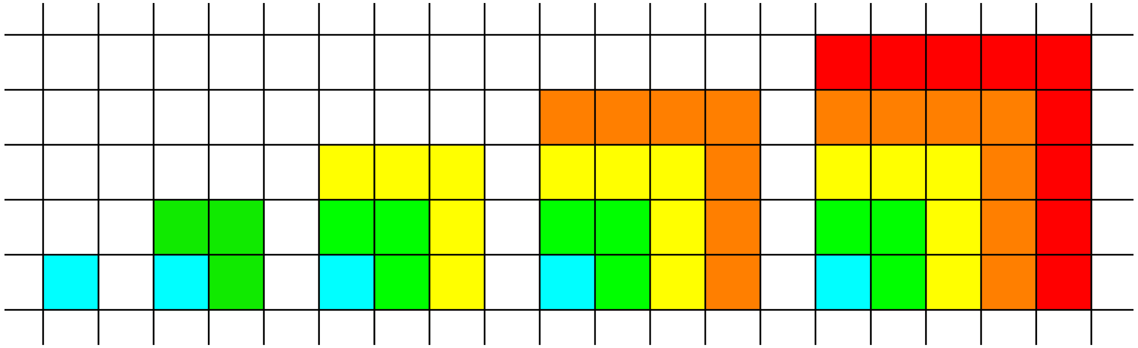
$$S:=\sum_{k=1}^{n}k$$

$$\begin{aligned}2S&=(\text{大三角形})+n\times 2\times(\text{小三角形})\\&=\tfrac{1}{2}(2n)\times n+n=n(n+1)\end{aligned}$$



たとえば・・・?!

$$1+3+5+7+\cdots+(2n-1)=\sum_{k=1}^{n}(2k-1)=n^2$$

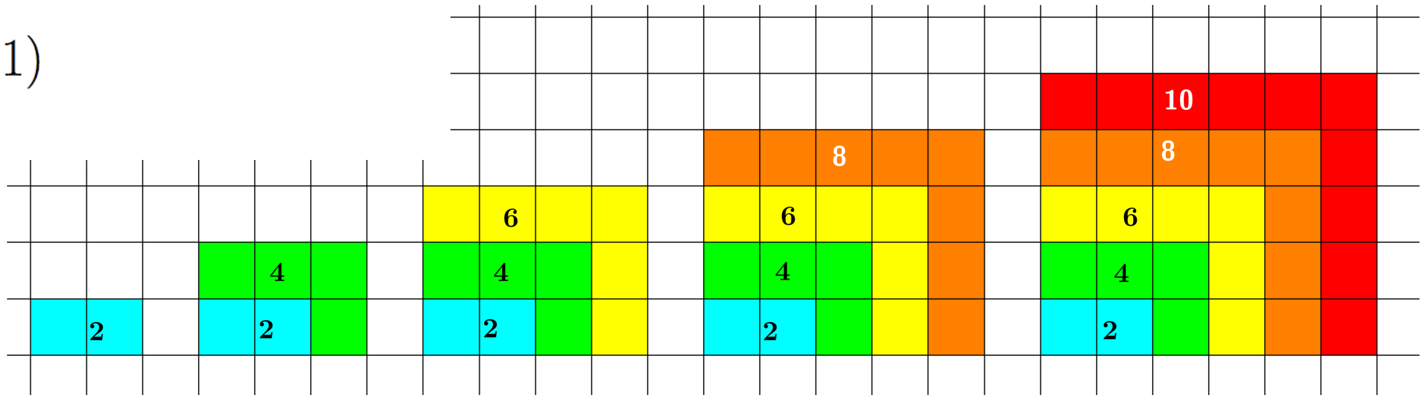


$$n^2=\sum_{k=1}^{n}(2k-1)=2\sum_{k=1}^{n}k-n$$

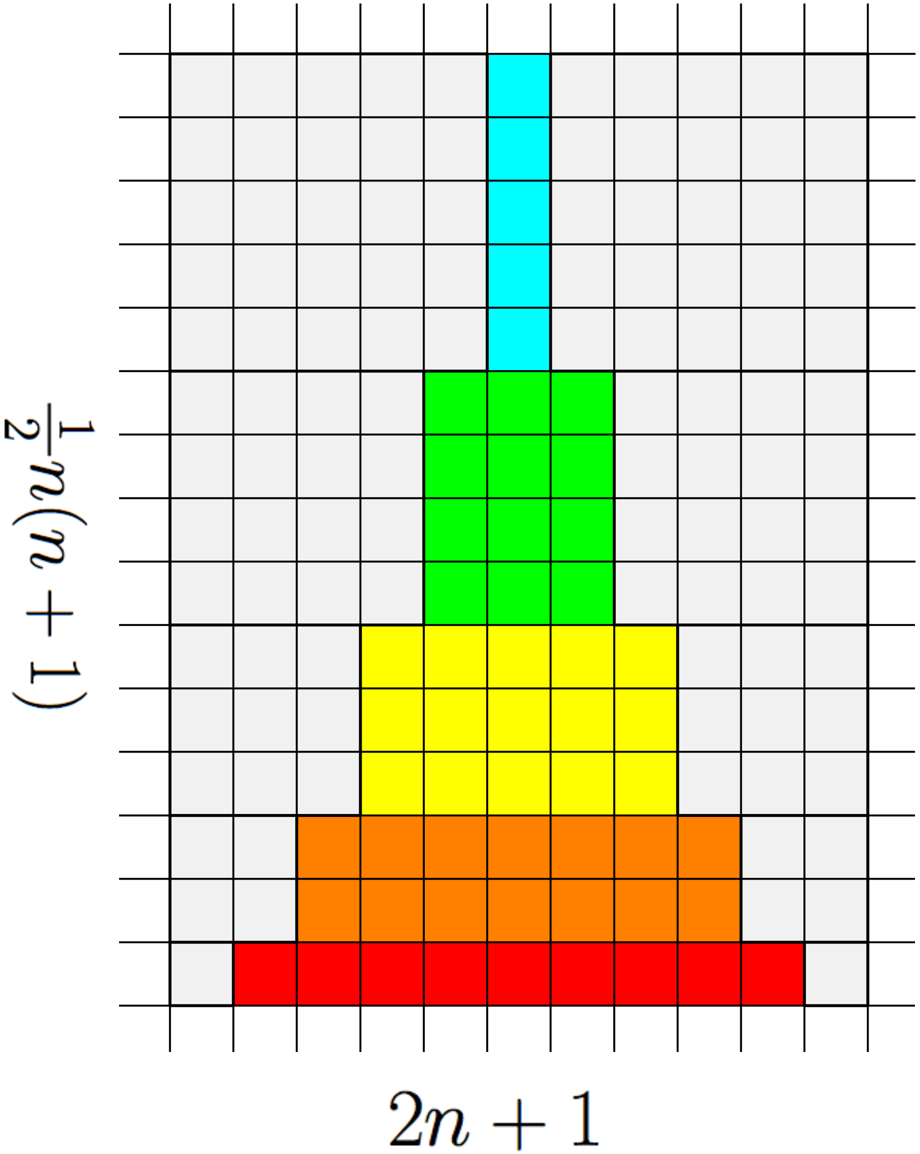
$$\therefore \quad \sum_{k=1}^{n}k=\frac{n^2+n}{2}$$

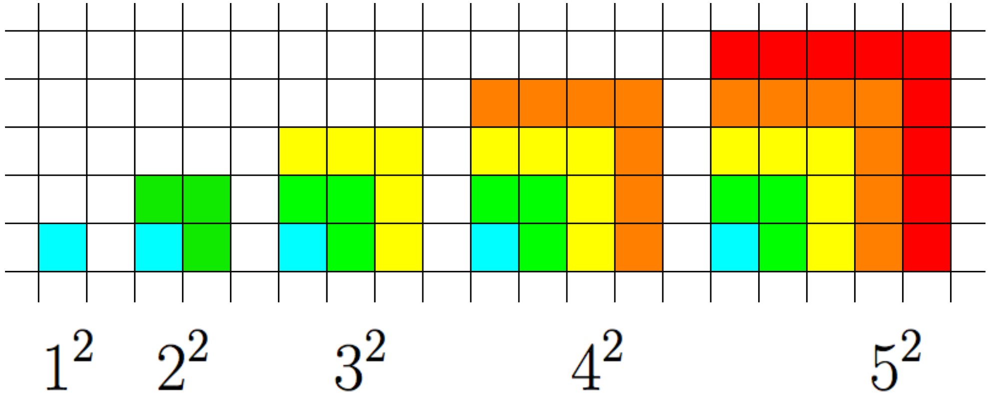
$$2+4+6+\cdots+2n=\sum_{k=1}^{n}(2k)=n\times(n+1)$$

$$\therefore \quad 2\sum_{k=1}^{n}k=n(n+1)$$



２乗和

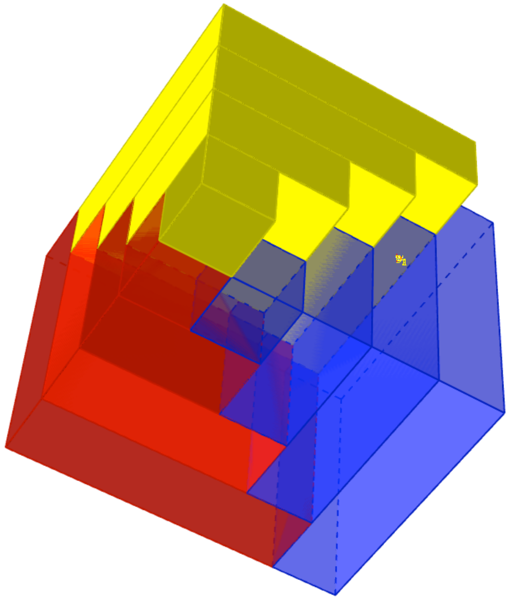


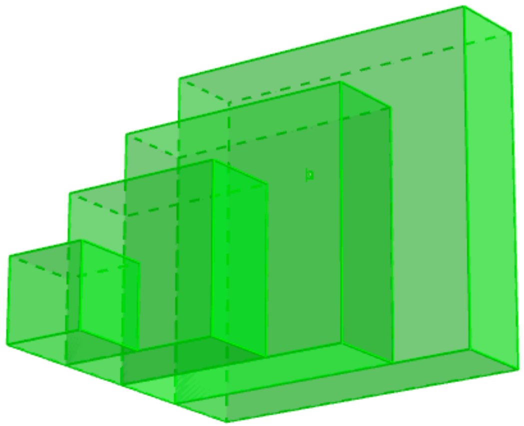


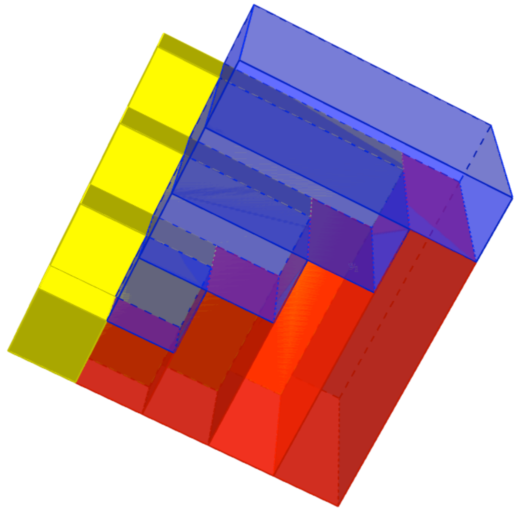
$$3 \times \sum_{k=1}^{n} k^2 = \frac{1}{2}n(n+1) \times (2n+1)$$

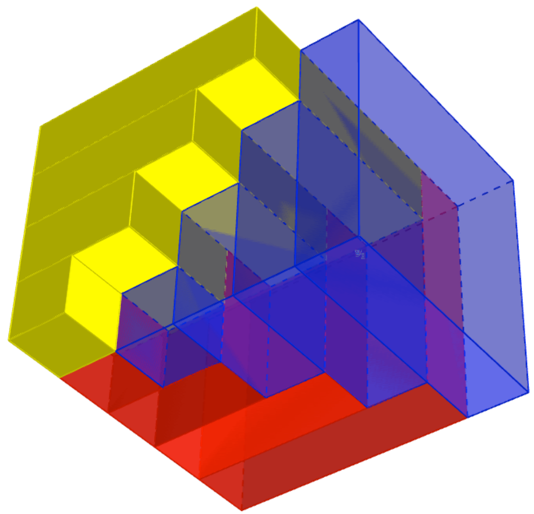
$$\therefore \quad \sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$$

２乗和 その２





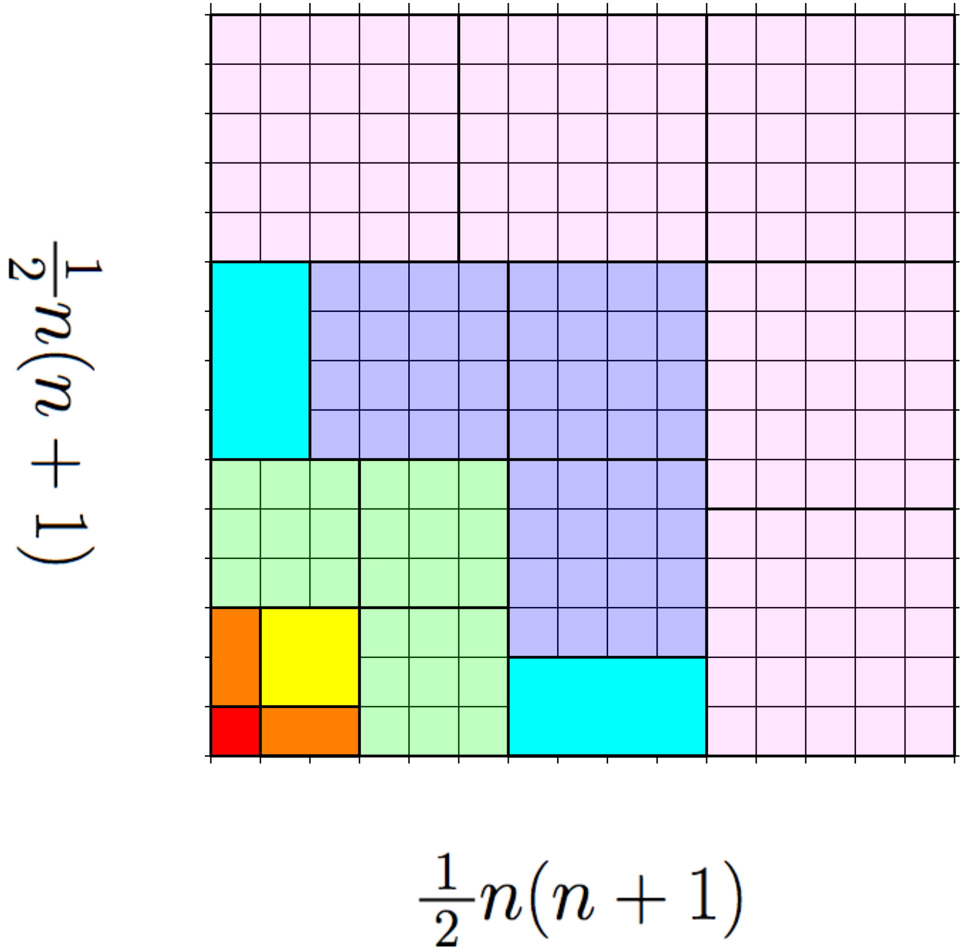




$$3 \times \sum_{k=1}^{n} k^2 = n \times n \times (n+1) + \frac{1}{2}n(n+1)$$

$$= \frac{1}{2}n(n+1)(2n+1)$$

3乗和 !!



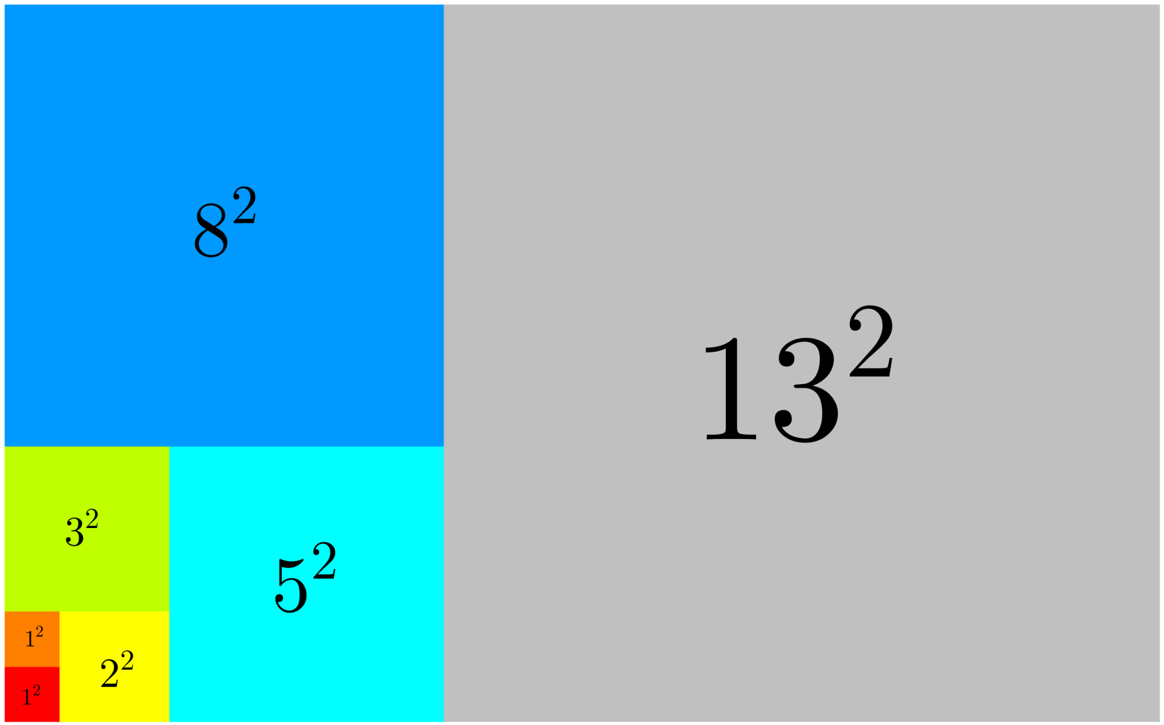
$$\sum_{k=1}^{n} k^3 = \left\{\frac{1}{2}n(n+1)\right\}^2$$

フィボナッチ数列

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ \cdots$$

$$F_{n+2} = F_{n+1} + F_n$$

$$F_1^2 + F_2^2 + F_3^2 + \cdots + F_n^2 = F_n F_{n+1}$$



レオナルド・フィボナッチ

Leonardo Fibonacci

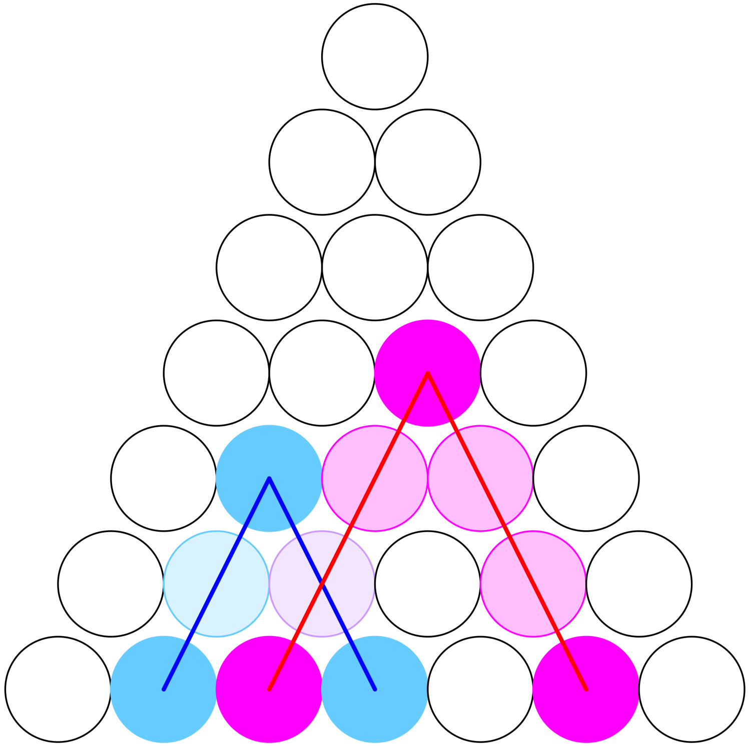
またの名を

ピサのレオナルド

Leonardo Pisano
(1170? ー1250?)

パスカルの三角形

$${}_{n}C_{2} = \frac{1}{2}n(n+1)$$



ブレーズ・パスカル

Blaise Pascal
(1623年ー1662)

歯車式計算機「パスカリーヌ」

https://ja.wikipedia.org/wiki/ブレーズ・パスカル

前半は

これでおしまい!!