AGU RT Seminar:

Workshop on Harmonic Analysis on Lie groups and Representations of Hecke Algebras

Date: 2023年3月20日(月) March 20th, Monday, 2023

Place: 青山学院大学理工学部 相模原キャンパス L603室 (L棟6階)
〒252-5258 相模原市 中央区 淵野辺 5-10-1
Room L603, Department of Mathematics, Aoyama Gakuin University

Speakers: Pavle Pandžić (Zagreb Univ.), 大島芳樹 (東京大学)
織田寛（拓殖大学）, 久保利久（龍谷大学）, 田内大渡（九州大学）

Organizer: 西山享（青山学院大学）, Kyo Nishiyama (AGU)

3/20, Mon, 9:30–10:00: Opening, discussions

3/20, Mon, 10:00–10:50: Toshihisa Kubo (Ryukoku Univ.)
The classification of the K-type formulas for the Heisenberg ultrahyperbolic equation of $\mathfrak{sl}(3,\mathbb{R})$

3/20, Mon, 11:10–12:00: Pavle Pandžić (Zagreb Univ.)
Dirac index and associated cycles of Harish-Chandra modules

3/20, Mon, 13:30–14:20: Yoshiki Oshima (Univ. Tokyo)
Discrete branching laws of derived functor modules

3/20, Mon, 14:40–15:25: Hiroshi Oda (Takushoku Univ.)
Inversion formula for Opdam–Cherednik transform associated with a root system of type BC

3/20, Mon, 15:45–16:30: Hiroshi Oda (Takushoku Univ.)
Functors connecting real reductive groups and graded Hecke algebras

3/20, Mon, 16:50–17:40: Taito Tauchi (Kyushu Univ.)
A generalization of the Kobayashi–Oshima uniformly bounded multiplicity theorem

3/20, Mon, 17:40–18:00: Closing, discussions

Abstract

Toshihisa Kubo (Ryukoku Univ.) [3/20, Mon, 10:00–10:50]

Title: *The classification of the K-type formulas for the Heisenberg ultrahyperbolic equation of $\mathfrak{sl}(3,\mathbb{R})$*

About ten years ago, Kable constructed a one-parameter family $\square_s^{(n)}$ $(s \in \mathbb{C})$ of differential operators for $\mathfrak{sl}(n,\mathbb{C})$ with $n \geq 3$. He referred to $\square_s^{(n)}$ as the Heisenberg ultrahyperbolic operator. In the viewpoint of intertwining operators, $\square_s^{(n)}$ can be thought of as an intertwining differential operator between certain parabolically induced representations for $\widetilde{SL}(n,\mathbb{R})$. In this talk we discuss about the classification of the K-type formulas of the space of K-finite solutions to the differential equation $\square_s^{(3)} f = 0$ for $\widetilde{SL}(3,\mathbb{R})$ and some related topics.

This is joint work with Bent Ørsted.

Pavle Pandžić (Zagreb Univ.) [3/20, Mon, 11:10–12:00]

Title: *Dirac index and associated cycles of Harish-Chandra modules*

We review the construction of Dirac operators, cohomology and index for Harish-Chandra modules, as well as associated varieties and associated cycles. Then we show how, for certain Harish-Chandra modules, the polynomial giving the dimension of the Dirac index of the corresponding coherent family can be expressed as an integer linear combination of the multiplicities in the characteristic cycle. This is joint work with S.Mehdi, D.Vogan and R.Zierau.

Yoshiki Oshima (Univ. Tokyo) [3/20, Mon, 13:30–14:20]

Title: *Discrete branching laws of derived functor modules*

We consider the restriction of Zuckerman's derived functor modules for symmetric pairs of real reductive groups assuming that it is discretely decomposable in the sense of Kobayashi. By a case-by-case argument, it can be shown that the restriction decomposes as a direct sum of Zuckerman's derived functor modules for the subgroup. In this talk, we would like to discuss how to obtain explicit branching formulas for some examples.

Hiroshi Oda (Takushoku Univ.) [3/20, Mon, 14:40–15:25]

Title: *Inversion formula for Opdam–Cherednik transform associated with a root system of type BC*

We give the inversion formula and the Plancherel formula for Opdam–Cherednik transform associated with a root system of type BC, when the multiplicity parameters are not necessarily nonnegative. This is a non-symmetric generalization of a recent result (DOI: 10.2969/jmsj/88728872) obtained with N. Shimeno and T. Honda.

Tatsuo HONDA, Hiroshi ODA, Nobukazu SHIMENO "Inversion formula for the hypergeometric Fourier transform associated with a root system of type BC," *Journal of the Mathematical Society of Japan*, J. Math. Soc. Japan Advance Publication, 1-37, (December, 2022)

Hiroshi Oda (Takushoku Univ.) [3/20, Mon, 15:45–16:30]

Title: *Functors connecting real reductive groups and graded Hecke algebras*

With a given reductive Lie group G there is an associated graded Hecke algebra $\boldsymbol{H}$. In order to connect representations of G and $\boldsymbol{H}$ we introduce the notion of radial pairs and give some examples. We also define a functor $\Xi : \operatorname{Rep}\boldsymbol{H} \to \operatorname{Rep}G$ such that $(\Xi(Y), Y)$ is a radial pair for $Y \in \operatorname{Rep}\boldsymbol{H}$. When G is in some series of classical groups, Ciubotaru and Trapa constructed a functor $F_{\mathrm{CT}} : \operatorname{Rep}G \to \operatorname{Rep}\boldsymbol{H}$. In this case, Ξ is a right inverse of F_{CT} and any radial pair (X, Y) satisfies $F_{\mathrm{CT}}(X) = Y$.

Taito Tauchi (Kyushu Univ.) [3/20, Mon, 16:50–17:40]

Title: *A generalization of the Kobayashi–Oshima uniformly bounded multiplicity theorem*

Let H be a real reductive algebraic subgroup of a real reductive algebraic group G and $G_{\mathbb{C}}$, $H_{\mathbb{C}}$ their complexifications. It was proved by T. Kobayashi and T. Oshima that the multiplicities of irreducible representations of G in the regular representation $C^{\infty}(G/H)$ are uniformly bounded if and only if $G_{\mathbb{C}}/H_{\mathbb{C}}$ is spherical, or equivalently, $H_{\mathbb{C}}$ acts on the full flag variety $G_{\mathbb{C}}/B$ with finitely many orbits, where B is a Borel subgroup of $G_{\mathbb{C}}$. Thus, there exists a relationship between the uniform boundedness of the multiplicities in the regular representation on G/H and the finiteness of the number of $H_{\mathbb{C}}$-orbits on the full flag variety $G_{\mathbb{C}}/B$.

In this talk, we discuss a variant of this relationship for a generalized flag variety $G_{\mathbb{C}}/Q_{\mathbb{C}}$, where Q is parabolic subgroup of G. More precisely, we prove that the multiplicities in $C^{\infty}(G/H)$ of degenerate principal series representations of G induced from Q are uniformly bounded if $H_{\mathbb{C}}$ acts on $G_{\mathbb{C}}/Q_{\mathbb{C}}$ with finitely many orbits using the theory of holonomic $\mathcal{D}$-modules.