

AGU 表現論セミナー

表現論とその周辺

— homaloidal 写像とクリフォード理論

Representation Theory and Related Topics: homaloidal maps and Clifford theory

Date: 2024 年 3 月 8 日 (金) March 8th Friday, 2024

Place: 青山学院大学理工学部 相模原キャンパス L416 室 (L 棟 4 階)
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世話人: 西山享 (青山学院大学), Kyo Nishiyama (AGU)

11:00 – 12:00: 西山享 (青学理工) Kyo Nishiyama (AGU)

実二重旗多様体の軌道とエルミート型グラスマン多様体

Real double flag varieties and Lagrangian-Grassmann variety of Hermitian type

14:00 – 15:00: 中島秀斗 (統数研) Hideto Nakashima (ISM)

ある非概均質的 homaloidal 多項式の b 関数の計算

A calculation of b -functions related to homaloidal polynomials arising from non-prehomogeneous group action

15:30 – 16:30: 小木曾岳義 (城西大) Takeyoshi Kogiso (Josai Univ.)

Open problem on polarizations of homaloidal polynomials with local functional equations

17:00 – 18:00: Pavle Pandžić (Zagreb Univeristy)

Clifford algebras, symmetric spaces and cohomology rings of Grassmannians

アブストラクト

- 西山亭（青学理工） Kyo Nishiyama (AGU)

実二重旗多様体の軌道とエルミート型グラスマン多様体

Real double flag varieties and Lagrangian-Grassmann variety of Hermitian type

Abstract: G を非コンパクト実簡約群として H をその対称部分群 (ある対合の固定部分群), $Q \subset H$, $P \subset G$ をそれぞれ放物型部分群とする. このとき H の作用する射影多様体 $X = H/Q \times G/P$ を実二重旗多様体という. 複素数体上の二重旗多様体の軌道の幾何学は Fresse と講演者の共同研究によって明らかになってきたが, 実数体上の二重旗多様体は複雑でまだ研究は緒に就いたばかりである. この講演では, $G = U(p, p)$ で $H = \mathrm{GL}_p(\mathbb{C})$ の場合に, おもに軌道の分類に関する結果を報告する. この場合には X/H は $\mathrm{GL}_q(\mathbb{C})$ ($0 \leq q \leq p$) の完全旗多様体上の $K_{\mathbb{C}}$ 軌道をすべて含むような有限個の軌道からなる構造を持っており, 興味深い.

この講演は田内大渡氏（青学理工）との共同研究に基づく.

Let G be a non-compact real reductive group and H its symmetric subgroup (the fixed point subgroup of a certain involution), and let $Q \subset H$, $P \subset G$ be respective parabolic subgroups. In this case, the projective variety $X = H/Q \times G/P$ with the diagonal action of H is called a *real double flag variety*. The geometry of the orbits of double flag varieties over the complex numbers has been studied extensively through joint research by Fresse and the speaker. However, the structure of orbits on double flag varieties over the real numbers is highly complicated and the study is still in its early stages. This talk will report on results mainly related to the classification of orbits in the case where $G = U(p, p)$ and $H = \mathrm{GL}_p(\mathbb{C})$. In this case, X/H consists of a finite number of orbits that include all the $K_{\mathbb{C}}$ orbits on the complete flag varieties of $\mathrm{GL}_q(\mathbb{C})$ ($0 \leq q \leq p$), which is interesting.

This talk is based on joint research with Taito Tauchi (AGU).

- 中島秀斗 (統数研) Hideto Nakashima (ISM)

ある非概均質的 homaloidal 多項式の b 関数の計算

A calculation of b -functions related to homaloidal polynomials arising from non-prehomogeneous group action

Abstract: 局所関数等式を満たす多項式の多くは概均質ベクトル空間の相対不変式になっているが、そうでないものも Faraut–Korányi (1994) や Kogiso–Sato (2016) などにより見つかっており、それらは homaloidal ($\text{grad log } p$ が双有理写像) という条件を満たしている。概均質ベクトル空間の相対不変式でないような homaloidal 多項式を探すのは意外に難しいが、最近そのような多項式が Hankel 行列のいくつかの成分を 0 としたものの行列式として現れることを発見した。本講演では、その多項式の系列について多変数 b 関数が具体的に計算可能であることを紹介したい。

本研究は城西大学の小木曾岳義氏との共同研究である。

Many polynomials satisfying local functional equations are relative invariants of prehomogeneous vector spaces, but it is known by Faraut–Korányi (1994) and Kogiso–Sato (2016) that there exist polynomials satisfying local functional equations which are not relative invariants of prehomogeneous vector spaces. All known examples of such polynomials satisfy a homaloidality condition, that is, the grad-log map is bi-rational. It is not easy to find homaloidal polynomials which are not relative invariants of prehomogeneous vector spaces, but recently we find a series of such polynomials related to Hankel matrices. In this talk, I would like to introduce these polynomials and give an explicit formula of b -functions of these polynomials. This is a joint work with Takeyoshi Kogiso.

- 小木曾岳義 (城西大) Takeyoshi Kogiso (Josai Univ.)

Open problem on polarizations of homaloidal polynomials with local functional equations

Abstract: In joint work with F.Sato, we study on local functional equation for polarization of homaloidal polynomial. We prove that the polarization of Prehomogeneous type polynomial is also Prehomogeneous type polynomial. However, it has not yet been proven that the polarization of non-Prehomogeneous type polynomial is also non-Prehomogeneous type polynomial and it is an open problem. In this talk, I will introduce one approach to this problem with examples.

- Pavle Pandžić (Zagreb Univeristy)

Clifford algebras, symmetric spaces and cohomology rings of Grassmannians

Abstract: We study various kinds of Grassmannians or Lagrangian Grassmannians over \mathbb{R} , \mathbb{C} or \mathbb{H} , all of which can be expressed as \mathbb{G}/\mathbb{P} where \mathbb{G} is a classical group and \mathbb{P} is a parabolic subgroup of \mathbb{G} with abelian unipotent radical. The same Grassmannians can also be realized as (classical) compact symmetric spaces G/K . We give explicit generators and relations for the de Rham cohomology rings of $\mathbb{G}/\mathbb{P} \cong G/K$. At the same time we describe certain filtered deformations of these rings, related to Clifford algebras and spin modules. While the cohomology rings are of our primary interest, the filtered setting of K -invariants in the Clifford algebra actually provides a more conceptual framework for the results we obtain. This is joint work with Kieran Calvert and Kyo Nishiyama.